1.5 Properties of a distribution function of a random vector

1. If $u_i < v_i$ then $F_{\mathbf{X}}(u_1, .., u_i, .., u_n) \leq F_{\mathbf{X}}(u_1, .., v_i, .., u_n)$ (*monotonicity*);

2. $\lim_{v_i \downarrow u_i} F_{\mathbf{X}}(u_1, .., v_i, .., u_n) = F_{\mathbf{X}}(u_1, .., u_i, .., u_n)$ (*right continuity*);

3. if some of $u_i \downarrow -\infty$, then $F_{\mathbf{X}}(u) \downarrow 0$;

4. if all $u_i \uparrow \infty$, then $F_{\mathbf{X}}(u) \uparrow 1$;

5.
$$F_{\mathbf{X}}(u_1, .., \infty, .., u_n) = P\left(\bigcap_{k \neq i}^{n} \{X_k \leq u_k\}\right);$$

6. $\Delta_{h_1} ... \Delta_{h_i} ... \Delta_{h_n} F_{\mathbf{X}}(u_1, .., u_i, .., u_n) \geq 0$, where $h_i \geq 0$ and

$$\Delta_{h_i} F_{\mathbf{X}}(u_1, .., u_i, .., u_n) = F_{\mathbf{X}}(u_1, .., u_i + h_i, .., u_n) - F_{\mathbf{X}}(u_1, .., u_i, .., u_n).$$

Any function in R^n which satisfies 1)-6) is a distribution function of some random vector

Example. *State with reasons whether the following functions are the distribution functions in* R^2*:*

1.

$$F(x,y) = \begin{cases} 0, & \textit{if} \quad x+y<0; \\ 1, & \textit{otherwise.} \end{cases}$$

2.

$$F(x,y) = \begin{cases} 0, & \textit{if} \quad x<0 \quad \textit{and} \quad y<0; \\ 1, & \textit{otherwise.} \end{cases}$$

3.

$$F(x,y) = \begin{cases} 0, & \textit{if} \quad x<0 \quad \textit{or} \quad y<0; \\ 1, & \textit{otherwise.} \end{cases}$$

Solution:

1. NO
2. NO
3. YES

□